

Exercise 5

Macroeconomics Exercise

2.02.2017

1

continuous Solow

- tech prog + pop. growth
- depreciation

① law of motion for $k = \frac{K}{AL}$

general form $F(K, L, A)$

conditions & implications of new + natural

$$\dot{K} = s \cdot \overset{\text{output}}{F(K, L, A)} - \delta \cdot K$$

law of motion for L & A

$$\dot{L} = n \cdot L \Rightarrow \frac{\dot{L}}{L} = n$$

$$\dot{A} = g \cdot A \Rightarrow \frac{\dot{A}}{A} = g$$

$$\dot{k} = \left(\frac{\dot{K}}{AL} \right) = \frac{d \left(\frac{K(t)}{A(t)} \cdot L(t) \right)}{d(t)}$$

$$= \frac{\dot{K} \cdot AL - K(\dot{A}L)}{(AL)^2}$$

$$= \frac{\dot{K} \cdot AL - K(\dot{A}L + A\dot{L})}{A^2 L^2}$$

$$= \frac{\dot{K}}{AL} - \frac{K}{AL} \cdot \frac{\dot{A}}{A} - \frac{K}{AL} \cdot \frac{\dot{L}}{L}$$

$$= \frac{\dot{K}}{AL} - k \cdot g + k \cdot n$$

$$= \frac{s \cdot F(k, AL) - \delta \cdot K}{AL} - kg - kn$$

$$= s \cdot f(k) - (n + g + \delta) k$$

compare to derivation of fundamental law of motion

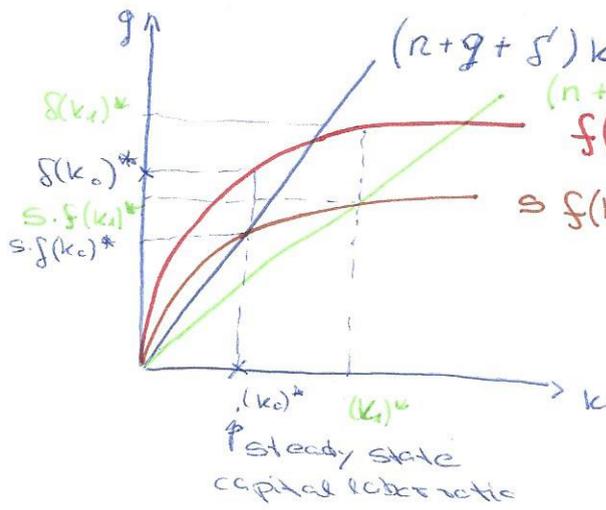
analog to discrete

Macroeconomics Exercise

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(2)

$g_1 > g_0$



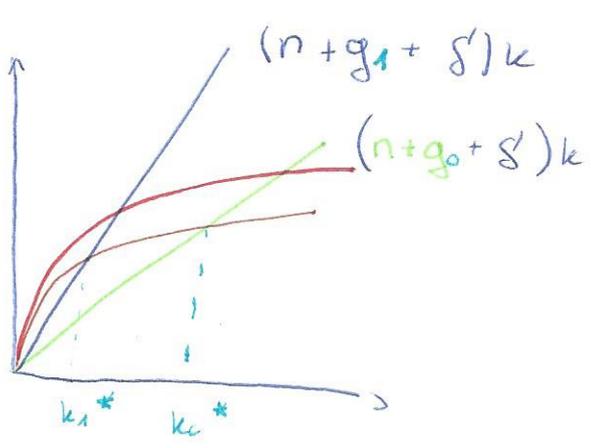
→ an increase in one of these parameters ⇒ increase in depr.

depreciation decreases $s'_0 > s'$
 new diminishing fct
 → flatter
 ⇒ higher savings & higher output

on balance growth path
 • no change in capital labor ratio
 $\dot{k} = 0 \Rightarrow s \cdot f(k)^* = (n + g + s') k^*$

increase in technological progress
 → diminishing fct turns upwards
 → opposite effect

$g_1 > g_0$

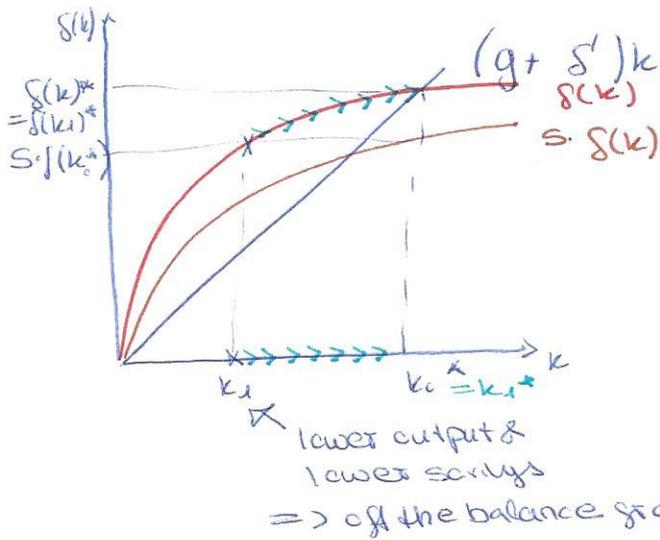


Macroeconomics
Exercise

Exercise 6

continuous slow world
one time jump

- tech prog
- no growth
- econ on b. growth path



we would have
on balance growth
path

$$s \cdot f(k_1^*) > (g + s')k$$

↑ larger than ↑ diminishing

→ move back
converging back
to balance
growth path

$$s \cdot f(k_1^*) \stackrel{?}{=} (g + s')k_1^*$$

$$Y = F(k, A \cdot L)$$

$$\dot{k} = s \cdot F(k, AL) - s'k$$

$$\dot{A} = g \cdot A$$

$$i = 0$$

$$\dot{k} = \frac{\dot{k}}{AL} = \frac{\dot{k}AL - k \cdot \dot{A}L - k \cdot A \dot{L}}{(AL)^2} \quad L=0$$

$$\Rightarrow \dot{k} = s \cdot f(k) - (g + s')k$$

$$3.) \quad s \cdot f(k) > (g + s')k \Rightarrow \dot{k} > 0 \quad \text{until } k^* = k_1^* = k_c^*$$

$$\text{since } y = f(k) \rightarrow y_1^* = f(k_1^*) = f(k_c^*) = y_c^*$$

change is only happening while getting back to balance growth path

Exercise 7

Golden Rule
maximize consumption in society

• max consumption
in balance growth path situation

production set were explicit!

⇒ Cobb Douglas

⇒ constant returns in scale

⇒ optimize production

$$Y(t) = K(t)^\alpha \cdot (A(t) \cdot L(t))^{1-\alpha} \quad 0 < \alpha < 1$$

① derive $y(t) = f(k(t))$

$$\frac{Y(t)}{A(t)L(t)} = \frac{K(t)^\alpha (A(t) \cdot L(t))^{1-\alpha}}{A(t) \cdot L(t)}$$

to the power of
-α ⇒ innumerate!

$$y(t) = \left(\frac{K(t)}{A(t) \cdot L(t)} \right)^\alpha$$

$$y(t) = k(t)^\alpha$$

per unit
output

② find k^* , y^* and c^* as fcts of s, n, g, s' and α

1.) $y(t) = k(t)^\alpha$

2.) analogue to ex. 5!

$$\dot{k}(t) = s \cdot f(k(t)) - (n+g+s')k(t)$$

3.) $\dot{L} = n \cdot L(t) \quad \dot{A} = g \cdot A(t)$

on the b.g.p.

$$\dot{k}(t) = 0$$

$$\Rightarrow s \cdot (k^*)^\alpha = (n+g+s')k^*$$

$$(k^*)^{\alpha-1} = \frac{(n+g+s')}{s}$$

$$(k^*)^{1-\alpha} = \frac{s}{n+g+s'}$$

$$\Rightarrow k^* = \left(\frac{s}{n+g+s'} \right)^{1/(1-\alpha)}$$

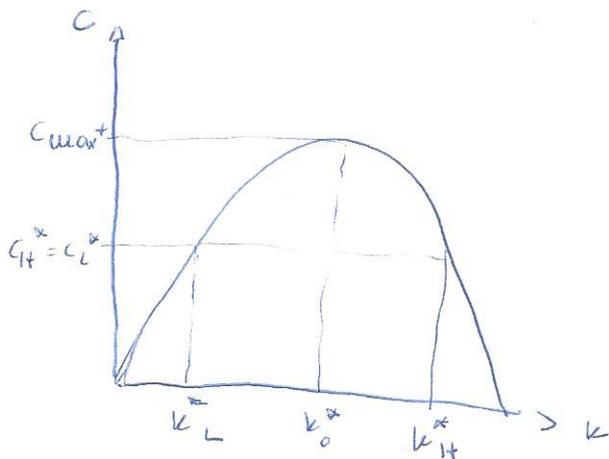
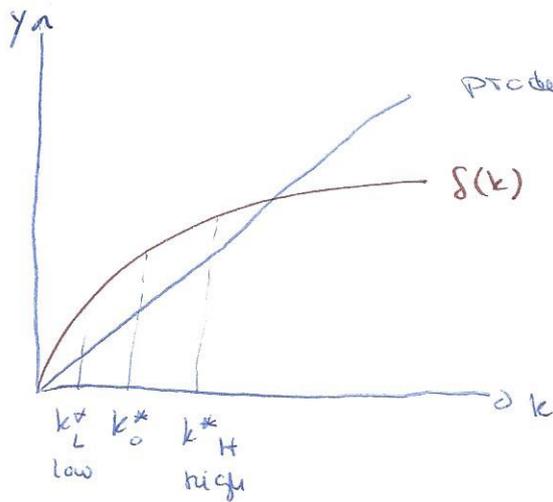
⇒ Action plan
what you have to
do in exercise
write down the
schemes as a
cook book!

$$y^* = f(k^*) = (k^*)^\alpha = \left(\frac{s}{n+g+s'} \right)^{\alpha/1-\alpha}$$

$$c^* = (1-s)y^*$$

$$c^* = (1-s) \left(\frac{s}{n+g+s'} \right)^{\alpha/1-\alpha}$$

b.g.p.
consumption



now look from a welfare perspective

- motive in society: —
interested in k^* & c^*

use equations for k^* & c^* :

use k^* first! & rearrange it for s

$$k^* = \left(\frac{s}{n+g+s'} \right)^{1/1-\alpha} \Leftrightarrow s = (n+g+s')(k^*)^{1-\alpha}$$

$$c^* = (1 - (n+g+s')(k^*)^{1-\alpha}) \left(\frac{(n+g+s')k^{*1-\alpha}}{(n+g+s')} \right)^{\alpha/1-\alpha}$$

$$c^* = (1 - (n+g+s')(k^*)^{1-\alpha}) \cdot k^{*\alpha}$$

$$= (k^*)^\alpha - (n+g+s') k^*$$

maximize c^*

$$\frac{\partial c^*}{\partial k^*} = \alpha (k^*)^{\alpha-1} - (n+g+s) = 0$$

condition for k_G^*
 $f'(k^*) = n+g+s'$

$$\alpha \cdot (k_G^*)^{\alpha-1} = (n+g+s')$$

$$\Rightarrow k_G^* = \left(\frac{\alpha}{n+g+s'} \right)^{1/(1-\alpha)}$$

4th part

s needed for k_G^*

$$k_G^* \text{ in: } s_G = (n+g+s')(k_G^*)^{1-\alpha}$$

$$= (n+g+s') \left(\left(\frac{\alpha}{n+g+s'} \right)^{1/(1-\alpha)} \right)^{1-\alpha}$$

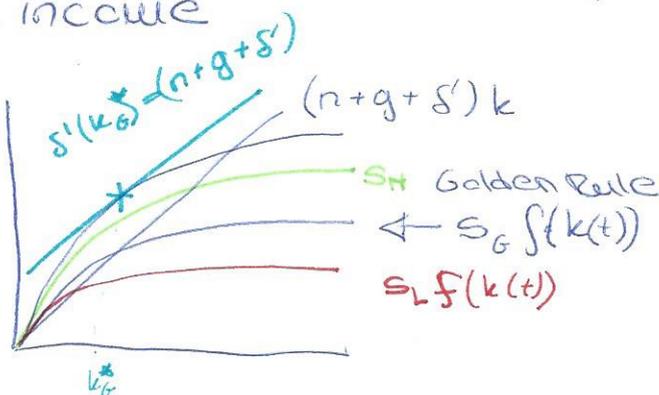
$$s_G = \alpha$$

savings rate

↖ exponent of capital term

CRS production fct
 α is the weight
 elasticity of output
 w.r.t. capital

with the underlying production fct, s needed for k_G^* equals output elasticity w.r.t. capital stock which in turn is equal to the capital share of income



Exercise 8

- even b.g.p.
- no tech prog
- decrease in pop. growth (e.g. in t_0)

1.) no tech. prog we can neglect A

$$L_0 A = 1 \quad \text{and} \quad \dot{A} = \sigma$$

$$Y(t) = F(K(t), L(t))$$

$$\dot{K}(t) = s \cdot F(K(t), L(t)) - \delta \cdot K(t)$$

until t_0 : $\dot{L}(t) = n_0 L(t)$

from t_0 : $\dot{L}(t) = n_1 L(t)$

$$k(t) = \frac{K(t)}{L(t)}$$

$$Y(t) = f(k(t))$$

$$\Rightarrow \text{l.o.m. } \dot{k}(t) = s \cdot f(k(t)) - (n + \delta) k(t)$$

graph is analogue to Ex. 2a) w/o $g: (n + \delta)k(t)$
turns rightward

after n decreased

$$s \cdot f(k_0^*) > (n_1 + \delta) k_0^* \Rightarrow \dot{k}(t) > 0$$

until b.g.p. (new)

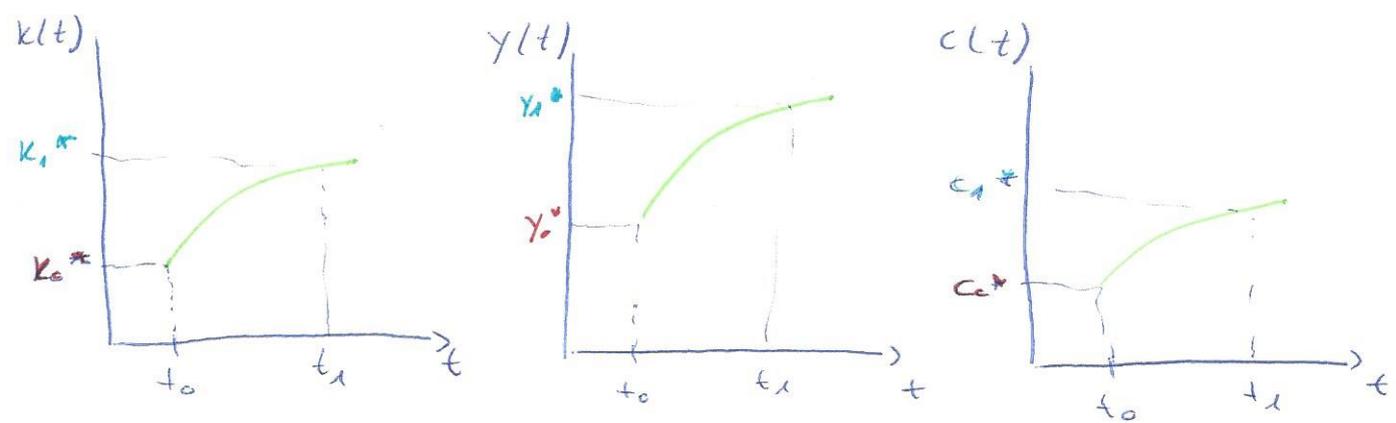
$$s \cdot f(k_1^*) = (n_1 + \delta) k_1^*$$

since $\dot{k}(t) > 0$ and $f'(k) > 0$

we know that $\dot{y}(t) > 0$

$$\text{since } c = (1-s)y \Rightarrow \dot{c} = (1-s)\dot{y} > 0$$

Exercise 8



2nd part => aggregate

Effect of a decrease in n on total output

$$y(t) = \frac{Y(t)}{L(t)}$$

$$\Rightarrow \dot{y}(t) = \dot{Y}(t) - \dot{L}(t)$$

$$\Rightarrow \frac{\dot{y}(t)}{y(t)} = \frac{\dot{Y}(t) - \dot{L}(t)}{Y(t) - L(t)}$$

2 additive components

$$= \underbrace{\frac{\dot{Y}(t)}{Y(t)}}_{\text{per capita } y(t)} - \underbrace{\frac{\dot{L}(t)}{L(t)}}_{=n}$$

initial b.g.p.

$$\frac{\dot{y}(t)}{y(t)} = \underbrace{\frac{\dot{Y}(t)}{Y(t)}}_{\sigma} + n_0 = n_0$$

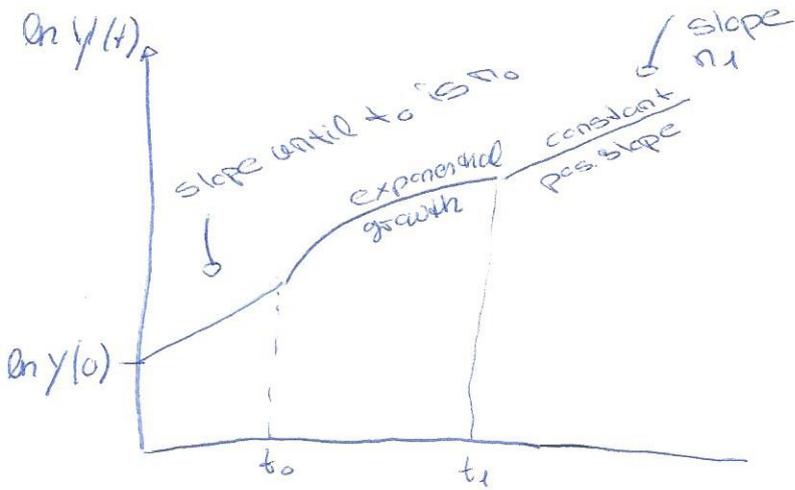
per capita output
-> what happens to
the aggregates

in new b.g.p. $\frac{\dot{y}(t)}{y(t)} = n_1 < n_0$

during the adjustment between the b.g.p. (from $t_0 \rightarrow t_1$)

$$n_0 < \frac{\dot{y}(t)}{y(t)} < n_1$$

$$\Rightarrow \frac{\dot{y}(t)}{y(t)} = \frac{d \ln y(t)}{dt}$$



start at $\ln y(0)$
what happens at t_0