The University of the Basque Country

Master in Economics EAP

Macroeconomics

IMPORTANT NOTES:

i) This problem set must be solved by work teams. All members of each team should be aware of the answer provided by his/her team: I might ask you to stop by my office to carefully explain the answer reported by your work team.

ii) Handing in instructions: ONE member (and only one member of your work team) should send a zipped file attached to an E-mail to me, cruz.echevarria@ehu.eus, **no later than November 28 at 23:00**. Late submissions will not be graded. The zipped file should be named `WT_Number_MACROECONOMICS_PS2.zip`, where "Number" should be conveniently substituted by the number assigned to your team, and contain all the files requested below. The "zip" file should contain *ii.a*) the Matlab files that you have used to obtain your numerical results, and *ii.b*) a "pdf" file with the answers to the questions asked below [please, indicate clearly the question that you are answering].

iii) You should be extremely careful with the formal aspect of your answer Matlab files, both script and function files. Indicate clearly the question that you are answering, and make sure that you use semicolons properly, so that the answer, and only the answer, is shown on the screen. *Please also make sure that the script generates the corresponding output file.*

iv) Last but not least. What does working in teams at this stage (*i.e.* when there is no ranking or competition among the members of the team) imply? Working in teams implies that *i*) **everyone** in the team must contribute with **their own answer** to the team, *and* that *ii*) at some time before the due date **all members in the team must meet together** and **understand** and **reach a unanimous agreement** about the common (team's) answer to hand in.

Pay-as-you-go Social Security system. Consider that the problem faced by a consumer born at time t were

$$\max_{\{c_{1t}, s_t, n_t, \ell_t, c_{2t+1}\}} U(c_{1t}, \ell_t, c_{2t+1}) = \ln c_{1t} + \theta \ln \ell_t + \beta \ln c_{2t+1}$$

$$\text{s. t.} \begin{cases} n_t + \ell_t = 1,\ n_t \geq 0,\ \ell_t \geq 0 \\ c_{1t} + s_t = w_t(1-\tau^{ss})n_t, \\ c_{2t+1} = (1+r_{t+1})s_t + b_{t+1}, \end{cases}$$

where $\theta > 0$, $\beta \in (0,1)$, and where the notation is similar to the one that we have used in the lectures with the novelty that ℓ_t denotes leisure time and that n_t denotes working time (i.e. labor supply) of a young individual of generation t. Note that young individuals choose how to split their total time endowment (normalized to 1) between leisure time, $\ell_t \in (0,1)$, and work time, $n_t \in (0,1)$. This requires a change in notation: denote the growth rate of population by m (instead of n which is reserved to denote young individuals' labor supply). The size of young population as of time t is denoted by N_t.

There are two new features in this problem compared to what you learned in the lectures. First, social security contributions are *proportional* to the wage income, the contribution rate being $\tau^{ss} \in (0,1)$. Second, we assume different preferences: labor supply is *elastic*. In other words, labor supply is a choice variable which may depend on the prices (not only on w_t, but also on r_{t+1}). More precisely, young individuals will choose to supply more or less labor (consume less or more leisure) depending on *i*) their taste for leisure relative to consumption, θ, *ii*) their time preference, β, *iii*) the net-of-tax wage rate, $w_t(1-\tau^{ss}) \equiv w_t^n$, and *iv*) the present value of their pension benefit, $b_{t+1}/(1+r_{t+1})$.

a) Solve (analytically) for the optimal choices: c_{1t}, c_{2t+1}, s_t, ℓ_t and n_t as functions of the preference parameters (θ and β), w_t^n and $b_{t+1}/(1+r_{t+1})$. You should simplify your answers as much as possible. In orther to check your answer, you should obtain the solution that we obtained in the lectures by setting now $\theta = 0$.

b) Obtain the equilibrium pension benefit at time $t+1$, b_{t+1}, under the alternative assumptions that the Social Security system follows *i*) a pay-as-you-go design, or *ii*) a fully funded scheme. page 19 & 20 OLG notes

c) The problem of the aggregate representative firm is the same that we have seen in the lectures page 6fff. OLG notes

$$\max_{\{Y_t,K_t,L_t\}} \pi_t = Y_t - L_t w_t - (r_t + \delta) K_t$$
$$\text{s.t. } Y_t = A K_t^{\alpha} L_t^{1-\alpha}$$

Find the first-order necessary conditions which characterize the demand for capital, K_t, and the demand for labor, L_t, and re-write them (the factor price equations) as functions of the capital-labor ratio, $k_t \equiv K_t/L_t$.

d) Write the condition which characterizes the equilibrium in the labor market at time t. Comment. *Think twice!!!* page 8fff. OLG notes

e) Solve for the equilibrium condition in the asset market *i*) under a pay-as-you-go system, and *ii*) under a fully funded system. You should obtain an expression which should depend only on s_t, τ^{ss}, w_t, k_{t+1}, m and $n_{t+1}(n_t)$. page 8fff. OLG notes page 10 ff. OLG notes & Exercise on OLG

f) SIMULATION OF THE STEADY STATE. The steady state is characterized by a constant stock of capital per unit of labor, k_*, a constant social security contribution, τ^{ss}, a constant pension benefit, b_*, and constant individuals' choice variables: c_{1*}, c_{2*}, s_*, n_* and ℓ_*. Assume, first, the following set of parameter values.

$$A = 5;\ \theta = 3.2;\ \beta = 0.6;\ \alpha = 0.3;\ m = 0.05;\ \delta = 0.1;\ \text{and } \tau^{ss} = 0$$

f.1) Find the steady state values for c_{1*}, c_{2*}, s_*, n_*, ℓ_*, k_* and $V_* \equiv U(c_{1*}, \ell_*, c_{2*})$, and fill in the cells in the following table:

τ^{ss}	c_{1*}	c_{2*}	s_*	n_*	ℓ_*	k_*	V_*
0							

f.2) Assume next that $\tau^{ss} = 0.35$, the other parameters being unchanged. Compute the steady state values for c_{1*}, c_{2*}, s_*, n_*, ℓ_*, k_*, b_* and V_*, and fill in cells in the following table under that alternative assumptions that the Social Security system follows i) a pay-as-you-go design, or ii) a fully funded scheme:

τ^{ss}	*SS scheme*	c_{2*}	s_*	n_*	ℓ_*	k_*	b_*	V_*
0.35	payg							
0.35	ff							

f.3) Plot the following 9 variables c_{1*}, c_{2*}, s_*, n_*, ℓ_*, k_*, b_*, V_* and the *generosity* of the pension system measured as the *gross replacement rate* which is defined as gross pension benefit (b_*) divided by gross pre-retirement earnings ($w_* \times n_*$) against τ^{ss}, for $\tau^{ss} \in [0, 0.35]$. In each of the 9 figures, you should plot the values obtained under both social security designs, i.e. i) pay-as-you-go and ii) fully funded

f.4) Based on the comparison between steady states (and, therefore, neglecting transitional dynamics), which would be your recommendation about the optimal social security scheme?